Physics of Neutrino Mass

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Main theme of the talk

☞ **Outline**

- HOW WELL DO WE UNDERSTAND THE NEUTRINO OBSERVATIONS ?
- WHAT HAVE WE LEARNED ABOUT NEW PHYSICS FROM THEM ?
- HOW CAN WE TEST THESE NEW IDEAS ?
- HOW DO THEY FIT INTO THE BIG PICTURE THAT WE MAY HAVE FOR OTHER PHYSICS e.g. GRAND UNIFICATION, COSMOLOGY etc ?

Outline of the three elctures

☞

1. Understanding small neutrino masses and mixings:
   - Seesaw Mechanism and some implications;
   - Mass matrices and Family Symmetries.
2. Origin of seesaw scale and implications for unification
   - Local B-L symmetry and left-right symmetric weak interactions;
   - Neutrino mass and grand unification:

   (i) Is SU(5) enough ?

   (ii) SO(10), a more suitable group and its predictions.
3. Neutrino mass related new physics
   - possible TeV scale W_R and Z'
   - light doubly charged Higgs;
   - Neutron-anti-neutron oscillation in reactors.

Summary of what we now know

(A. de Gouvea and B. Kayser's talks for details)

☞ Solar + Atmospheric + KamLand + K2K

- MIXINGS Defined as:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{\alpha i} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

- where $U_{PMNS} =$

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & c_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} K$$

- $K \;=\; diag(1, e^{i\phi_1}, e^{i\phi_2})$
- SOLAR: $sin^2\theta_{12} \simeq 0.31 \pm 0.02$
- ATMOS: $tan^2\theta_{23} \simeq 0.89^{+0.31}_{-0.21}$
- REACTOR: $\theta_{13} \leq 0.23$

MASSES

☞ **We only know mass two difference squares**

➢ ATMOS: $|\Delta m^2_{13}| = (2.6 \pm 0.2) \times 10^{-3}$ eV^2; (1σ)

➢ SOLAR: $\Delta m^2_{21} \;=\; (7.9 \pm 0.3) \times 10^{-5}$ eV^2 ; (1σ)

➢ *MASS PATTERN STILL UNKNOWN*

☞ **Possibilities**

1. ::NORMAL::$\longrightarrow m_1 \ll m_2 \ll m_3$
   $\longrightarrow \Delta m^2_{31} > 0$; $m_3 \simeq 0.05$ eV; $m_2 \simeq 0.009$ eV
   In particular, in this case $\frac{m_3}{m_2} \sim 6$;
   What is m_1 ?
2. ::INVERTED::$\longrightarrow m_1 \simeq m_2 \gg m_3$
   $\longrightarrow \Delta m^2_{31} < 0$; $m_1 \simeq m_2 \simeq 0.05$ eV
3. ::DEGENERATE:: $m_1 \simeq m_2 \simeq m_3 \longrightarrow \Delta m^2_{31} > or < 0$

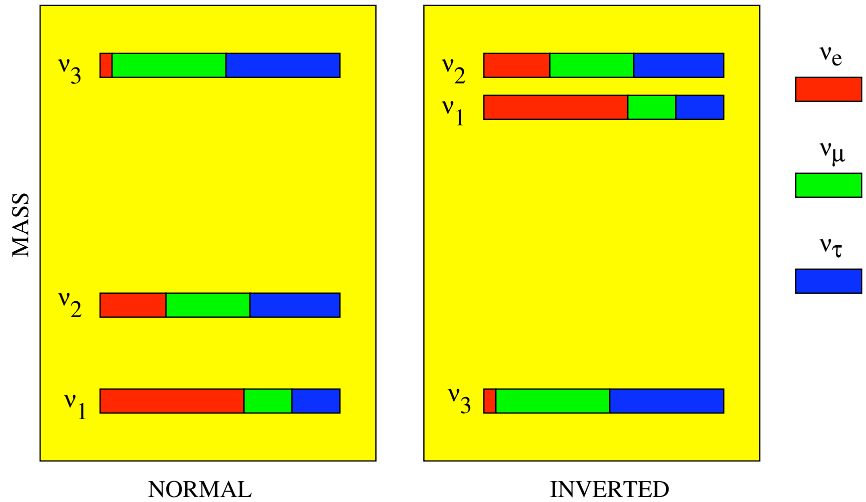


Figure 1: Two possible pattern of masses; Compare with quarks for which $m_{u,d} \ll m_{c,s} \ll m_{t,b}$

☞ **Compare with quarks for which** $m_{u,d} \ll m_{c,s} \ll m_{t,b}$ **and** $\theta^q_{13} \simeq .004$**;** $\theta^q_{23} \simeq 0.04$ **and** $\theta^q_{12} \simeq 0.22$

☞ **Another fundamental property of the neutrino that we stll do not know is:**

Is neutrino its own antiparticle ? i.e. is $\nu = \bar{\nu}$

If $\nu = \bar{\nu}$, it is Majorana; otherwise Dirac

For Majorana neutrino, we have

$$2N \rightarrow 2P + 2e^-$$

i.e. neutrinoless double beta decay; amplitude directly proportional to neutrino Majorana mass

Overall mass scale

☞ **We need to know the lightest mass in Case (i) (normal) and (ii)(inverted) and absolute mass in case(iii)**

: ☞ **Experimental Information**

1. ^3H Decay end point: $\Sigma_i m_i^2 |U_{ei}|^2 \leq 2.2$ eV^2 (KATRIN expected to improve it to 0.2 eV)

2. Cosmology: $\Sigma m_i \leq 0.4$ eV (WMAP, SDSS: will be improved by Planck)

3. If neutrino Majorana i.e. $\nu = \bar{\nu}$, $\beta\beta_{o\nu}$ results imply: $\Sigma_i U_{ei}^2 m_i \leq 0.3 - 0.5$ eV (Expected improvement to 0.03 eV)

How many neutrinos?

☞

- Z-width measurement at LEP and SLC implies three ν's coupling to Z (active neutrinos $\nu_{e,\mu,\tau}$)

- Most persuasive argument for sterile neutrinos is the oscillation results from LSND: negative results by MiniBooNe can still be made consistent with LSND, if there are two sterile neutrinos with specific masses: $\Delta m^2_{41} = 0.66$ eV^2 and $|U_{e4}U_{\mu 4}| = 0.044$; e.g.$\Delta m^2_{51} = 1.44$ eV^2 and $|U_{e5}U_{\mu 5}| = 0.022$ (Maltoni and Schwetz (07)).

- Severe constraints on the number of ν_s from BBN (≤ 0.3 ;

- Also severe constraint on the number from structure formation; (≤ 2.5 at 68% c.l.) (Hannestad review, 2006)

WHERE WE ARE HEADED ?

☞ **(i) Dirac vrs Majorana in $\beta\beta_{0\nu}$ decay search:**

Expts: MAJORANA (Ge$^7 6$), EXO(Xe), CUORE (Te)...

(ii) Normal vrs Inverted hierarchy:

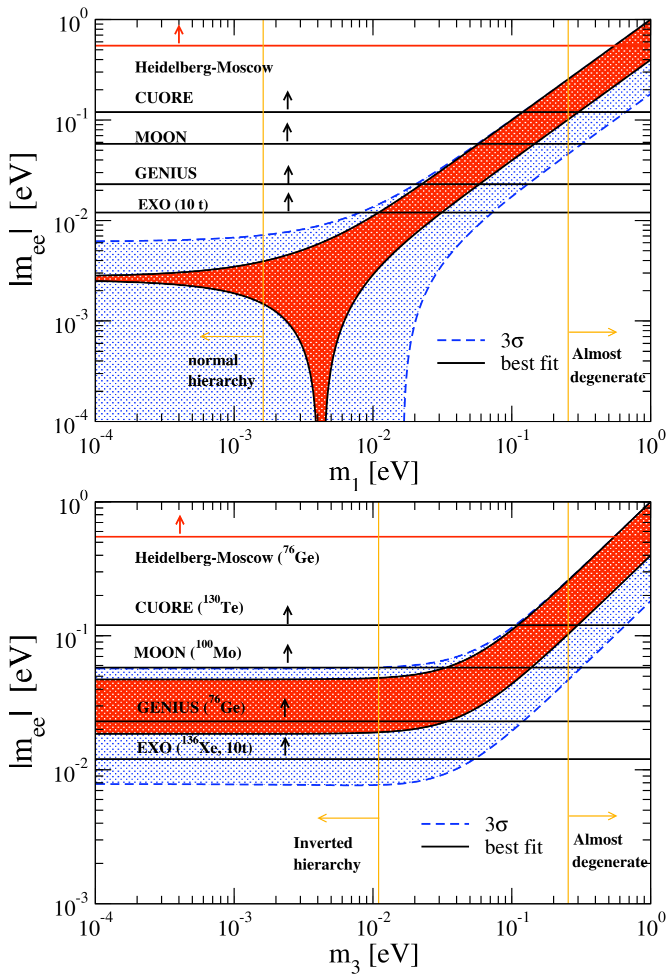
Long baseline neutrino experiments- JPARC, NoVa,

(iii) Determining θ_{13}:

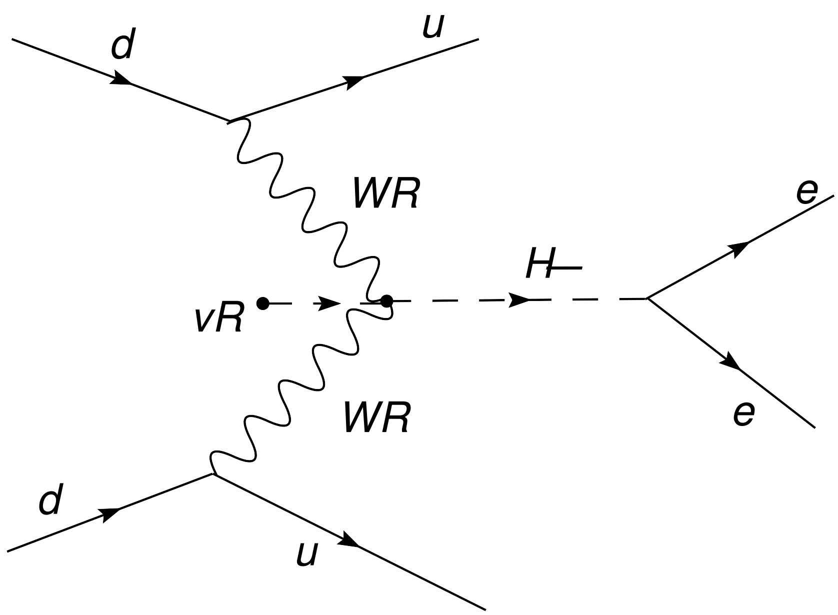
Reactor experiments: Double CHOOZ, Daya Bay
$\theta_{13} \leq 0.05$ **will reveal important new symmetries:**

Testing inverted hierarchy in $\beta\beta_{0\nu}$ decay searches:

☞ $< m_{\beta\beta} > \simeq \Sigma_i U_{ei}^2 m_i$: **leads in the case of inverted hierarchy to a lower bound if neutrino is Majorana.**



☞ **CAUTION! even with normal hierarchy, there could be "large" effects from heavy particles such as sparticles, doubly charged Higgs bosons or RH Majorana neutrinos.**



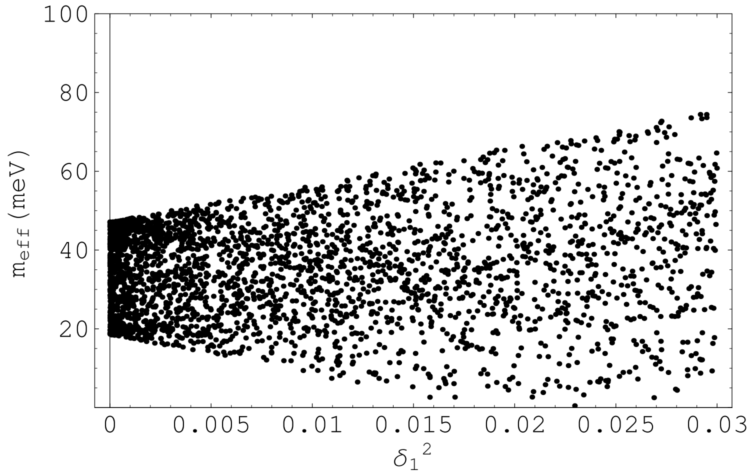


Figure 2: Scatter plot of $|m_{eff}| \equiv < m_{ee} >$ in the case of inverted hierarchy in the presence of a sterile neutrino with mass ~ 1 eV as a function of δ_1^2. Note that the minimum value of $|m_{eff}|$ depends on the best fit value for the LSND mixing parameter in a 3+1 scheme. The scatter of points is due to the variation of the phases α and β between 0 to π.

Similarly presence of sterile neutrinos could mask the inverse hierarchy effects!

Prospects for discriminating between Dirac and Majorana neutrino

☞ **Sign of Δm^2, $\beta\beta_{0\nu}$ and KATRIN result can tell us a lot:**

$\beta\beta_{0\nu}$	Δm^2_{32}	KATRIN	Conclusion
yes	> 0	yes	Degenerate, Majorana
yes	> 0	No	Degenerate, Majorana or normal or heavy exchange
yes	< 0	no	Inverted, Majorana
yes	< 0	yes	Degenerate, Majorana
no	> 0	no	Normal, Dirac or Majorana
no	< 0	no	Dirac
no	< 0	yes	Dirac
no	> 0	yes	Dirac

Neutrino Mass and new physics

☞ Challenges for theory:

➢ Why $m_\nu \ll m_{u,d,e}$?

➢ Why are neutrino mixings so much larger than quark mixings ?

➢ Why is $\frac{\Delta m^2_\odot}{\Delta m^2_A} \ll 1$ but $\gg \left(\frac{m_\mu}{m_\tau}\right)^2$?

➢ What do neutrinos tell us about physics beyond the standard model e.g. new symmetries; any new forces ?

➢ How do neutrinos fit into the big picture of grand unification,cosmology etc.

A Primer on Fermion masses and mixings

☞ **Look for bilinears of the form $\bar{\psi}_L\psi_R$ in the Lagrangian**

If there are more fermions of the same kind, then
$\mathcal{L}_{mass} = M_{ab}\bar{\psi}_{a,L}\psi_{b,R}$

☞ **Masses and mixings from the Lagrangian**

➢ M_{ab}= Mass matrix

➢ Diagonalize the mass matrix
$U^\dagger MV = diag(m_1, m_2, \cdot, \cdot)$

➢ U, V gives the mixings between different (L, R) fermions, ψ_a and m_i are the actual masses e.g. for quarks, U_{ab} contains the CKM mixings (e.g. $U_{CKM} = U_u^\dagger U_d$, where$U$ and V denote the rotations in the up and the down sector)

Oscillations

☞

➢ The "flavor" eigenstate :$|a> \quad = \quad \Sigma_i U_{ai}|i>$ and as it evolves with time, we have oscillations.

➢ Key to understanding masses and mixings is clearly the form of the mass matrix

☞ DIRAC vrs MAJORANA MASS

- Lorentz Invariance allows two kinds of mass terms for fermions: $\bar{\psi}_L\psi_R$ or $\psi_L^T C^{-1}\psi_L$ (or $L \leftrightarrow$ R)

- Note under a symmetry transformation $\psi \to e^{i\alpha}\psi$, the first mass is invariant whereas the second term is not;

- Fermions only with the first kind of mass are called Dirac fermions and those with both kinds are called Majorana fermions

- Dirac fermion unlike Majorana requires an extra symmetry: e.g. for $e, \mu, q..$, extra symmetry is $U(1)_{em}$; since $Q(\nu) = 0$, no such symmetry is there for ν

- Hence for neutrinos, Majorana-ness is more natural; also small mass is easier for Majorana neutrino.

- For Majorana mass, $\nu = \bar{\nu}$

Neutrino Majorana mass matrices

☞ **If there are no RH or SM singlet neutrinos, the Majorana mass matrix is symmetric and has 6 real entries and three phases- 9 parameters;**

Physical observables: $m_i; \theta_{ij}; \delta^D; \alpha^M_{1,2}$**;**

Measured observables: 4;
Can have mass matrices with three real parameters describing observations- but entries at suitable places;

e.g. $\mathcal{M}_\nu = \begin{pmatrix} 0 & a & b \\ a & 0 & c \\ b & c & 0 \end{pmatrix}$ **is already ruled out by solar observations;**

An example of a minimal mass matrix that is allowed is:

$$\mathcal{M}_\nu = \sqrt{\Delta m^2_A}\begin{pmatrix} \epsilon & b\epsilon & b\epsilon \\ b\epsilon & 1+\epsilon & b\epsilon - 1 \\ b\epsilon & b\epsilon - 1 & 1+\epsilon \end{pmatrix}$$

☞ **A four parameter example:** $\mathcal{M}_\nu = \begin{pmatrix} d & a & b \\ a & 0 & c \\ b & c & 0 \end{pmatrix}$;

leads to inverted hierarchy and observable neutrino-less double beta decay.

Standard model

☞ Details

➢ Gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$

➢ Matter: Doublets: $Q \equiv \begin{pmatrix} u_L \\ d_L \end{pmatrix}$; $\psi_L \equiv \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$;

Singlets: u_R; d_R; e_R

Higgs: $H \equiv \begin{pmatrix} H^0 \\ H^- \end{pmatrix}$

➢ $\mathcal{L}_Y = h_u \bar{Q}_L H u_R + h_d \bar{Q}_L \tilde{H} d_R + h_e \bar{\psi}_L \tilde{H} e_R + h.c.$

Fermion masses

☞ **Masses arise from symmetry breaking** $< H^0 >= v_{wk}$

- ➢ $\mathcal{L}_m \;=\; \bar{u}_{a,L} M^u_{ab} u_{b,R} + \bar{d}_{a,L} M^d_{ab} d_{b,R} + \bar{e}_{a,L} M^e_{ab} e_{b,R}$;
- ➢ gives masses to quarks and charged leptons only
- ➢ Therefore $m_\nu = 0$ in the standard model.
- ➢ No effect of the standard model can give mass to the neutrino since B-L is a good symmetry of the standard model.

☞ **Could there be some source of B-L breaking hidden in the standard model that would give** $m_\nu \neq 0$ **?**

Could it be gravity ?

☞ **Standard model + gravity**

➢ global symmetries could be broken by nonperturbative gravitational effects such as black holes or worm holes etc.

➢ if so, they could induce B-L breaking operators into standard model e.g. $(\psi_L H)^2/M_{P\ell}$;

➢ They lead to $m_\nu \simeq \frac{v^2_{wk}}{M_{P\ell}} \sim 10^{-5}$ eV- clearly too small to explain atmospheric neutrino deficit. The Answer is: NO!

☞ : NEUTRINO MASS AND NEW PHYSICS: MECHANISMS AND MODELS

Nu-standard model

☞ **Simplest possibility: Add ν_R to the standard model**

➢ New term in the $\mathcal{L}_Y$: $h_\nu^\dagger \bar{\psi}_L H \nu_R + h.c.$
Could it be that this is the source of neutrino mass when symmetry breaks ?

➢ In principle it could but this is very unnatural because of the following reasons:
(i) since $m_\nu \leq 0.1$ eV, this will mean $h_\nu \sim 10^{-12}$? why is it so small ?

(ii) This Yukawa coupling then is likely to be very similar to the quarks in which case it will be hard to understand why
(a) the neutrino mixings are so different from quark mixings and
(b) why $\frac{m_2}{m_3} \sim \frac{1}{6}$ for neutrinos whereas for up quarks the same number is 1/100 and for down quarks it is 1/40-50 ?

Seesaw mechanism

☞

- ➢ RH neutrinos are different from the rest of the standard model particles- since they are SM singlets unlike all others.
- ➢ Because of this gauge invariance allows a new term that can be added to the SM i.e. $\frac{1}{2}M_R\nu_R^T C^{-1}\nu_R + h.c.$

  Important point: M_R breaks B-L symmetry
- ➢ This makes neutrino masses different from the masses of other SM fermions:

Seesaw diagram

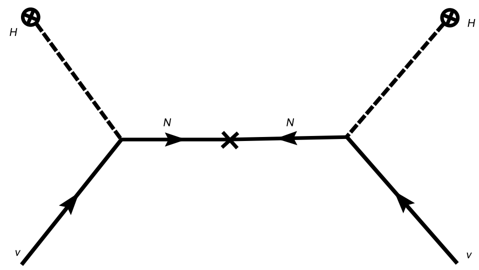


Figure 3: Seesaw diagram

☞ **Leads to Majorana neutrinos.**

$\longrightarrow$ **Mass matrix for** (ν_L, ν_R) **system:**

$\begin{pmatrix} 0 & h_\nu v \\ h_\nu^T v & M_R \end{pmatrix}$ $M_R \gg h_\nu v$, **mass eigenvalues: heavy :**

$\longrightarrow$**:** M_R **and light:** $\mathcal{M}_\nu \simeq -v_{wk}^2 h_\nu^T M_R^{-1} h_\nu$**;**

Roughly $m_\nu \simeq -\frac{h_\nu^2 v^2}{M_R}$. **Since** $M_R \gg v_{wk}$, **this explains why** $m_{\nu_i} \ll m_{u,d,e...}$. **(Type I seesaw)**

Minkowski (77); Gell-Mann, Ramond, Slansky; Yanagida; Glashow; R. N. M., Senjanovic (1979)

Detailed derivation of seesaw formula

☞ $\mathcal{L}_{\nu,mass} \;=\; h_{\nu,ij} v_{wk} \bar{\nu}_R \nu_L + \frac{1}{2} \nu_R^T M_{R,ij} \nu_R \;+\; h.c.$;

Write $\nu \;=\; \begin{pmatrix} \xi \\ i\sigma_2 \chi^* \end{pmatrix}$;
ξ, χ **two-component objects;**

γ **matrix convention:** $\gamma_i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}$; $\gamma_0 \;=\; \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$;
$\gamma_5 \;=\; \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$;
$\nu_L \;=\; \begin{pmatrix} \xi \\ 0 \end{pmatrix}$ **and** $\nu_R \;=\; \begin{pmatrix} 0 \\ i\sigma_2 \chi^* \end{pmatrix}$;
$$\mathcal{L}_{\nu,mass} \;=\; -\, i \begin{pmatrix} \xi^T & \chi^T \end{pmatrix} \begin{pmatrix} 0 & h_\nu v_{wk} \\ h_\nu^T v_{wk} & M_R \end{pmatrix} \begin{pmatrix} \xi \\ \chi \end{pmatrix} \;+\; h.c.$$

(Same as in the previous page.)

Higgs triplets and Type II seesaw

☞ **Instead of adding the ν_R, add a Higgs triplets, Δ_L with $Y = 2$**

i.e. $\Delta_L = \begin{pmatrix} \Delta^{++} \\ \Delta^{+} \\ \Delta^{0} \end{pmatrix}$**;**

$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$**.**

The new contribution to Yukawa coupling:

$\mathcal{L}'_Y = f_T L^T C^{-1} \vec{\tau} \cdot \vec{\Delta}_L L + h.c.$**;**

Gives a term: $\nu_L^T C^{-1} \Delta_0 \nu_L$**;**

If $< \Delta_L^0 > \sim$ eV, then we understand small neutrino mass.

The big question is:why $< \Delta_L^0 > \ll v_{wk}$?

: TYPE II SEESAW.

Other variations

☞

➢ Fermionic seesaw: Add a triplet fermion $\vec{F}$ which then allows a new Yukawa of the form $\bar{L}\vec{\tau}H \cdot \vec{F}$;

➢ Give a large mass to $\vec{F}$;

➢ TYPE III SEESAW;
Diagram similar to type I seesaw.

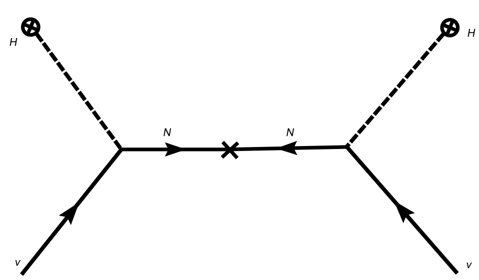


Figure 4: Seesaw diagram

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Alternatives to seesaw and ultralightness of Dirac ν's

☞ **Two classes of alternatives to seesaw:**

(i) Loop models- populate weak scale with many new particles e.g. singly and doubly charged bosons- not motivated by any other physics; then one can generate two loop Majorana masses that are small.

(ii) Extra dimensions: Brane-bulk models for fundamental forces and forces;
in such models, SM particles are in the brane and RH neutrinos in the bulk. The overlap of the wave function is given by M/M_P where M is the multi-TeV string scale- this is a new mechanism for understanding the smallness of neutrinos:

Neutrinos are Dirac particles and are accompanied by an infinite tower of RH neutrinos.
Such models are highly constrained by cosmology and astrophysics.

Derivation of neutrino mixings

☞ $\mathcal{L}_m(\nu, e) \ = \ \frac{1}{2}\nu_L^T C^{-1}\mathcal{M}^\nu \nu_L + \bar{e}_L M^e e_R + h.c.$

Diagonalize:

$$U_\nu^T \mathcal{M}^\nu U_\nu = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}; U_\ell^\dagger M^e V_\ell = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}$$

Neutrino mixing matrix $U_{PMNS} \ = \ U_\ell^\dagger U_\nu$

Note that observed mixings encoded in U_{PMNS} depends on both the charged lepton as well as neutrino mass matrices !

This fact is important while trying to understand neutrino mixings.

Is there any way to separately measure U_ℓ ?
Not if weak,em and gravity are the only interactions of leptons.
Could be possible for example if there are lepto-quark type interactions or even supersymmetry under certain circumstances !

☞ : IMPLICATIONS OF HIGH SCALE TYPE I SEESAW

How many right handed neutrinos for seesaw ?

☞ **One RH neutrino seesaw does not work- it leads to two massless light neutrinos!!**

Either two or three RH neutrinos can give realistic models !!

☞ **Advantages and disadvantages of 2 vs 3 case**

➢ 3 ν_R's: (i) # of parameters =18 (too many since possible inputs from nu-mass physics=9)
(ii) Complete quark-lepton symmetry- theoretically appealing.

➢ 2 ν_R's: (i) # of param. = 12;
(ii) however, theoretically less appealing;
(iii) Predicts one massless neutrino.

What is the Seesaw Scale ?

☞ **Three theory motivated benchmark values (for 3 RH nus):**

•: **In GUT theories,** $M_u \sim M_D$ **and hence** $m_{D,33} \sim m_t$;

$\sqrt{\Delta m_A^2} \sim \frac{m_t^2}{M_R}$ **gives** $M_R \sim 10^{14}$ **GeV- near GUT scale.**
Could m_ν **be the first indication of grand unification ?**

If we give up the GUT hypothesis, m_D **is free and other possibilities arise !!**

•: $M_R \simeq 10^{11}$ **GeV if** $m_{D,33} \sim m_\tau$;

Also emerges in theories as $M_R \simeq \sqrt{M_W M_P}$

•: $M_R \simeq$ **TeV if Dirac mass suppressed due to symmetries:**
Effects observable at LHC (see later).

High Scale Seesaw

☞ **Most well motivated due to the way it fits into many other discussed schemes of beyond the standard model physics e.g.**

•: **Simple Grand Unification (see Lecture II);**

•: **Cosmology e.g. origin of matter, inflation etc.**

Testing High Scale type I Seesaw

☞ **Testing seesaw with Lepton Flavor Violation**
In Non-SUSY ν_R extended standard model (ν-SM) with small m_ν due to high scale seesaw, rare lepton decays e.g. $\mu \to e + \gamma$ are highly suppressed.

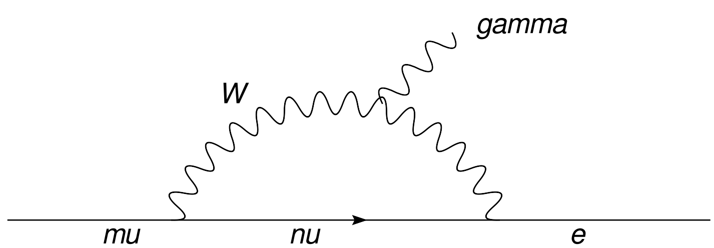


Figure 5: $\mu \to e + \gamma$ in non-SUSY nu-SM

☞ $A(\mu \to e + \gamma) \simeq \frac{eG_F m_\mu m_e m_\nu^2}{\pi^2 m_W^2} \mu_B$
leads to $B(\mu \to e + \gamma) \sim 10^{-50}$**.**
(Same with other such LFV processes)

Present experimental situation

☞ $B(\mu \to e + \gamma) \leq 1.4 \times 10^{-11}$ **and MEG expt to push it by at least three orders of magnitude;**

$B(\tau \to \mu + \gamma) \leq 4.5 \times 10^{-8}$**: Belle;**
$B(\tau \to e + \gamma) \leq 1.2 \times 10^{-8}$**.**

LFV in high scale SUSY seesaw models

review and references: Masiero, Vives and Vempati (2004)

☞ **Large neutrino mixings can induce significant $\tau \to \mu + \gamma$ and $\mu \to e + \gamma$.**

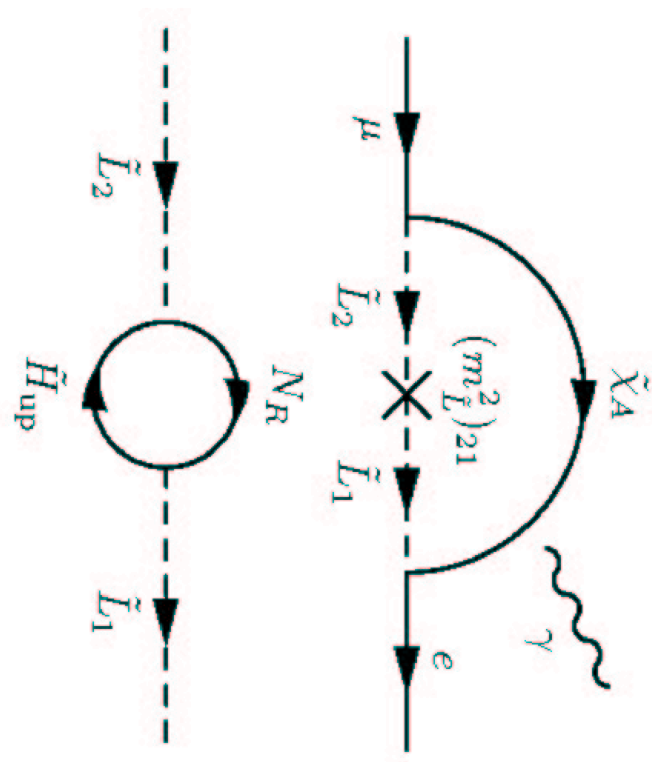
With Seesaw + supersymmetry $\to$, superpartners remember high scale effects; large lepton mixings, radiatively inducing large 23 and 12 slepton mixing, and hence significant $B(\tau \to \mu + \gamma)$ and $B(\mu \to e + \gamma)$.

Predictions for LFV

☞ **Formula:**

$$B(\mu \to e + \gamma) \;=\; \frac{\alpha^3}{G_F^2 M_S^8}\left|\frac{(3+a_0^2)m_0^2}{8\pi^2}\right|^2 \left|\Sigma\, h^\dagger_{\nu,\mu k} ln\left(\frac{M_U}{M_k}\right) h_{\nu,ke}\right|^2$$

Typical SUSY models: $M_S^8 \simeq 0.5 m_0^2 m_{1/2}^2 (m_0^2 + 0.6 m_{1/2}^2)^2$



Predictions for LFV

☞ **Predictions depend on supersymmetry parameters e.g. $m_{1/2}, m_0, A, tan\beta$ and μ as well as on seesaw scale M_R and the Dirac coupling h_ν.**
GUT models can fix M_R and h_ν leading to definite predictions for LFV effects(see later). Typical expectations are between $10^{-11} - 10^{-14}$ for $B(\mu \to e + \gamma)$ and upto 10^{-7} for $B(\tau \to \mu + \gamma)$

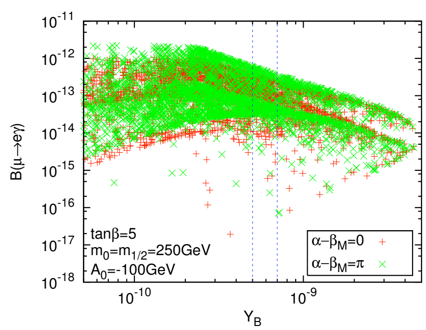
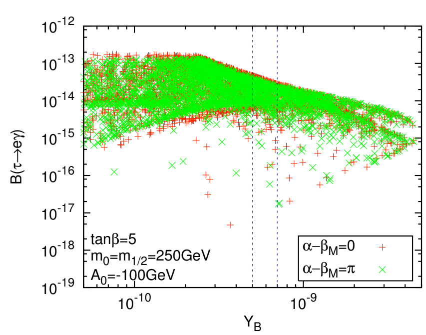


Figure 6: **Figures from Petcov, Rodejohann,Takanishi, Shindou; demonstrates LFV-leptogenesis connection**



Seesaw, Leptogenesis and origin of matter

See lectures by Plumacher

- **why is $\frac{n_B - n\bar{B}}{n_\gamma} \simeq 10^{-10}$?**
  - If there is CP violation in the lepton sector (in particular in RH neutrino couplings), then
  - $\Gamma(N_R \rightarrow \ell + H) - \Gamma(N_R \rightarrow \bar{\ell} + H) \neq 0 \rightarrow$ lepton asymmetry;
  - baryon violation at the weak scale converts the lepton asymmetry into baryon asymmetry.
  - Same physics used for generating small neutrino masses.

Fukugita, Yanagida, 1984

- **Seesaw cosmologically appealing**

Dependence of Leptogenesis on Seesaw parameters

☞ **Formula for baryon asymmetry: Y_B:**

$Y_B \;=\; -10^{-2}\kappa\epsilon_\ell$;
κ: **wash-out factor;**
ϵ_ℓ **is lepton asymmetry and is dictated by seesaw theory.**

$$\epsilon_\ell \simeq -\frac{3}{8\pi}\frac{1}{(h_\nu h_\nu^\dagger)_{11}} Im\left[(h_\nu h_\nu^\dagger)^2_{21}\right]\frac{M_1}{M_2}$$

☞ **Key point to notice is that Y_B depends on $h_\nu h_\nu^\dagger$ whereas LFV depend on $h_\nu^\dagger h_\nu$.**

Parameterizing Seesaw

☞ **Seesaw formula:** $\mathcal{M}_\nu \;=\; -v_{wk}^2 h_\nu^T M_R^{-1} h_\nu$**:**

This formula can be inverted:

$$h_\nu(M_R) \;=\; \frac{1}{v_{wk}}\sqrt{M_R^d}R\sqrt{M_\nu^d}U^\dagger_{PMNS}$$

Casas, Ibarra (2001)

☞ **where** $R^TR = 1$ **i.e. a complex symmetric orthogonal matrix:**

Parameter counting:

$6(inR) + 6(masses) + 6(anglesandphases) \;= 18$**;**

Parameterizing R**:** $R \;=\; R_{12}R_{23}R_{31}$ **where**

$$R_{12} \;=\; \begin{pmatrix} cosz_1 & sinz_1 & 0 \\ -sinz_1 & cosz_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ **and similarly for others.**

: A consequence of this is: lepton asymmetry is independent of PMNS phases observable at low energies whereas LFV depends on that.

In actual models however, they are linked !

Lepton edms also as tests of Seesaw, leptogenesis

☞ **In leptogenesis models, CP violation will "sip" down from high scale to the neutrino mixings via the seesaw mechanism and can give rise to effects such electron and muon edm.**

EDM predictions in generic seesaw models for leptogenesis

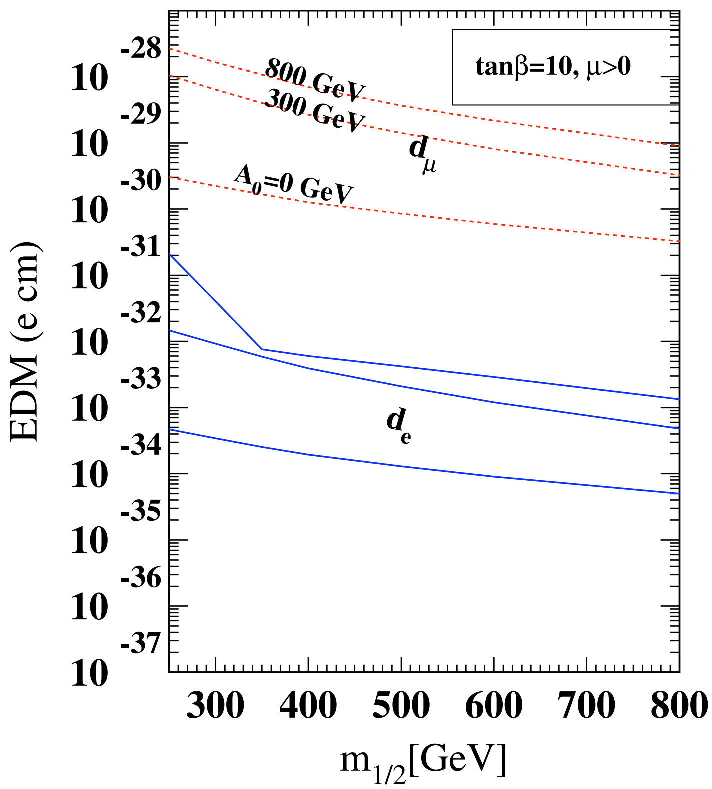


Figure 7: Typical values of electron and muon edm in seesaw models that explain the origin of matter: (Dutta and RNM, 2003)

☞ **Present expt limits and future possibilities**

d_e **: Now:** $\leq 7 \times 10^{-28}$ **ecm; Future:** 10^{-32}-10^{-35} **ecm;**

d_μ **: Now:** $\leq 3.7 \times 10^{-19}$ **ecm; Future:** $10^{-24} - 10^{-26}$ **ecm;**

Radiative corrections from high scale Seesaw

☞ • **Seesaw mechanism is a high scale effect- its predictions are therefore high scale predictions !Measurements are made at the weak scale; so how to calculate the weak scale values ?**

Babu, Leung, Pantaleone; Chankowski, Plucciniek (93); Antusch, Kersten, Lindner and Ratz

☞ $\frac{d\mathcal{M}_\nu}{dt} = C_l h_l^\dagger h_l \mathcal{M}_\nu + \mathcal{M}_\nu C_l h_l^\dagger h_l + \alpha \mathcal{M}_\nu$

$$\mathcal{M}_\nu(\mu) = I_C(\mu) R(\mu) \mathcal{M}_\nu(m_Z) R(\mu).$$

$R \approx diag(1, 1, Z_\tau(\mu)), \qquad Z_\tau - 1 = C_l \frac{h_\tau^2}{16\pi^2} \log \frac{\mu}{m_Z} \equiv \frac{m_\tau^2 \tan^2\beta}{8\pi^2 v_{wk}^2} \log \frac{\mu}{m_Z},$

SM: $\mu = 10^{10}$ **GeV,** $(Z_\tau - 1)_{SM} \simeq 10^{-5}$**.**

MSSM: $(Z_\tau - 1)_{MSSM} \approx (Z_\tau - 1)_{SM} \tan^2\beta$ **For** $\tan\beta = 50$**,** $Z_\tau - 1 \sim 0.03$ **(significant!)**

Typical effect on mixings

☞ **e.g on θ_{13}:**

$$\theta_{i3}(M_Z) \simeq$$
$$\theta_{13}(M_R) \ + \ (Z_\tau - 1)\frac{1}{4} sin2\theta_{12} sin2\theta_{23}\left[\frac{m_2+m_3}{m_2-m_3} + \frac{m_1+m_3}{m_1-m_3}\right]$$

(i) Normal hierarchy: effect very small i.e. $\delta\theta_{13} \leq 0.01$ **!!**

(ii) For inverted or degenerate case, effect can be large !

An interesting application

Parida, Rajasekaran, RNM (2003)

☞ **It could be that just like there is grand unification of forces, there could be unification of quarks with leptons, possibly implying that at the seesaw scale (or GUT scale),**

$\theta^q_{ij} \ = \ \theta^\nu_{ij}$.

What then are the values of neutrino mixings at the weak scale ?

This time look at θ_{12}:

$\dot{\theta}_{12} \approx -A \sin 2\theta_{12} \sin^2\theta_{23} \frac{|m_1+m_2 e^{\phi_{12}}|^2}{\Delta m^2_{21}}$

θ_{13} **case given earlier:**

For normal hierarchy, this is a small effect. But for degenerate neutrinos, $m_{1,2} \gg \Delta m^2_{12}$; hence there is a big enhancement and large lepton mixings can arise from small lepton mixings at seesaw scale.

An actual computation

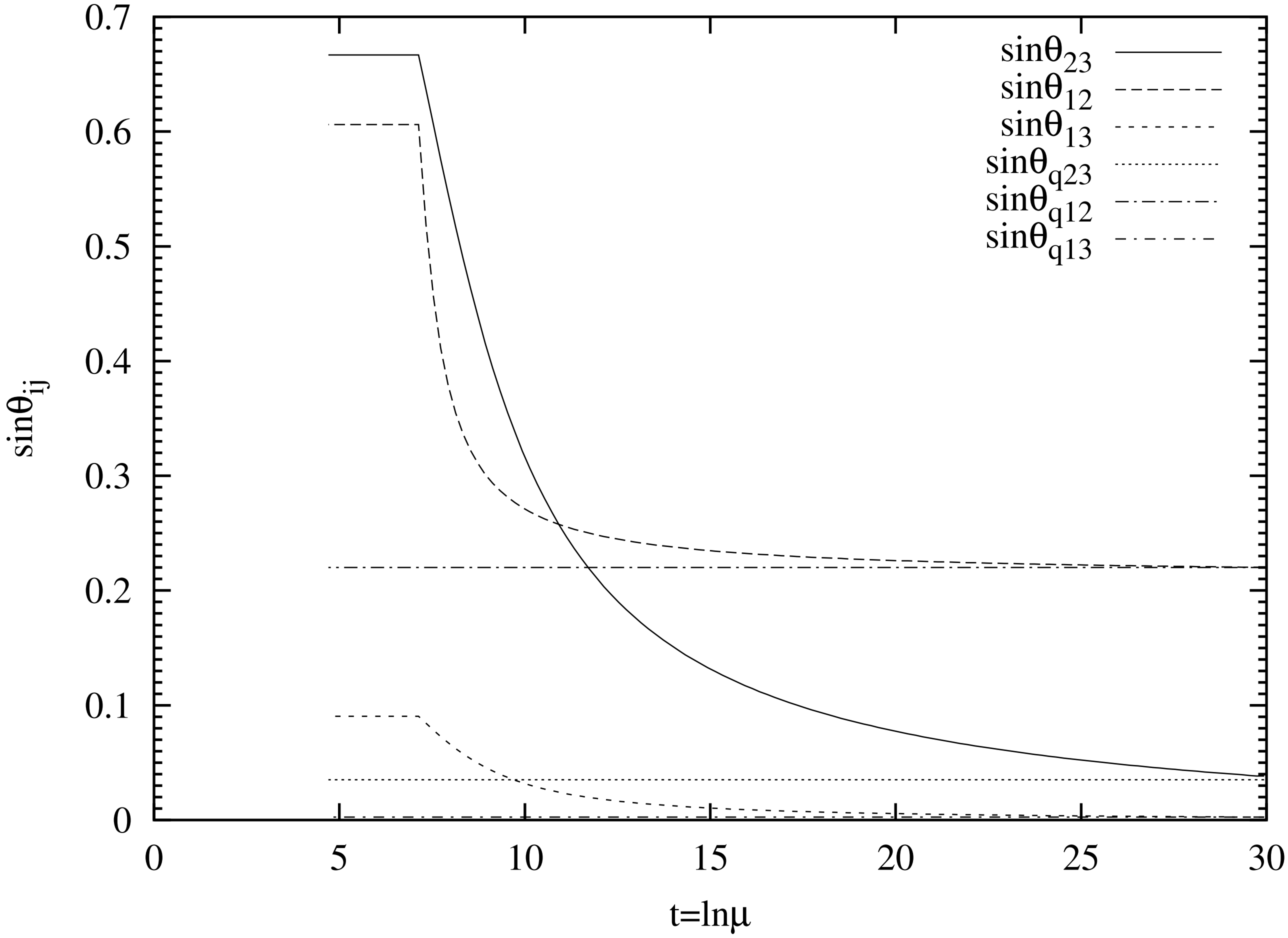


Figure 8: Large lepton mixings from small high scale mixings

Can we ever determine all the seesaw parameters ?

☞ **In principle, the answer is yes !**

Case (i): 3×3 seesaw:

:
Measure the following:
(a): three m_{ν_i}, three mixings and three phases [9 inputs];

(b) leptogenesis: [1 input];

(c) $B(\mu \to e + \gamma)$; $B(\tau \to (e, \mu) + \gamma)$: [3 inputs]

(d) CP violating parameters: μ, e, τ edms; [3 inputs]

(e): CP violating asymmetries in $\mu \to e + \gamma$; $\tau \to (e, \mu) + \gamma$ decays: [3 inputs]

One more than needed.

Case of 3×2 seesaw

☞ **A more realistic possibility**
(a): Two m_{ν_i}, three mixings and one phase [6 inputs];

(b) leptogenesis: [1 input];

(c) $B(\mu \rightarrow e + \gamma)$; $B(\tau \rightarrow (e, \mu) + \gamma)$: [3 inputs]

(d) CP violating parameters: μ, e edms; [2 inputs]